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A Handbook for Mathematics Majors

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A Message from the Chair

Mathematics matters now more than ever. Every technical career field is influenced by data and the need to interpret and understand it. A training in mathematics gives you not only a thorough working knowledge of the relevant analytical tools and techniques, but also instills in the successful student the ability to confront, deconstruct, and solve any problem placed in front of them. These are all qualities that employers like to see in their job candidates.

At UMW we take the liberal arts approach to our mathematics curriculum. As a mathematics major you will choose from a wide selection of high-quality courses in many different branches of contemporary mathematics. In addition to single courses, you will have the opportunity to complete rigorous year-long sequences in areas such as abstract algebra, numerical analysis, probability and statistical inference, and real analysis. You will see connections and applications to other disciplines as well as the deep relations between (seemingly) unrelated branches of mathematics. Along the way, your communication skills will be strengthened through Writing Intensive and Speaking Intensive courses.

You can gain all of the aforementioned skills and knowledge by taking enough classes to complete the major, thereby earning your B.S. in mathematics. But guess what? Across the country there are thousands of mathematics majors doing the same thing. What are you going to do to set yourself apart, to enhance your credentials, to make employers seek you out? There is a wealth of opportunities for such enhancements here at UMW, but you have to get involved to get the most out of them. We have independent studies, honors projects, summer undergraduate research (with pay!), internships, and more. We the faculty want to help you advance your studies through such programs, but we can’t unless you tell us that you want to pursue them. When the time comes to ask for recommendation letters for that big job, you want to be more to us than that student who came to class and took really good notes. The solution? Get involved!

There are tons of opportunities here at UMW and in the Department of Mathematics; many of them are described in the following pages. Every faculty member in the Department of Mathematics took his or her job because they love working with students. When you find something that interests you, speak up and pursue it. We’re all here for the same reason: to make the most out of your four years of study at UMW. Reach out if you have any questions.

Randall D. Helmstutler, chair
The Mission of the Department

The University of Mary Washington Mathematics Department is dedicated to offering the highest quality mathematics education in a student-centered environment focused on liberal learning. The Mathematics Department aims to provide our students with the quantitative, critical thinking, and communication skills needed to succeed in their majors, in their careers, and in their lives.

Accordingly, the Mathematics Department is committed to meeting the educational and professional needs of our students. Our mathematics faculty develop close mentoring relationships with our students through small classes, undergraduate research projects, independent study experiences, internships at local agencies, and our local chapters of the Mathematical Association of America and the national honorary society Pi Mu Epsilon. The Mathematics Department also promotes the many connections that can be made between mathematics, other disciplines, and the community around us.
1 Introduction

1.1 The Study of Mathematics

If you are like most incoming students, you are familiar with mathematics in the sense of algebra, geometry, and calculus. For instance, at this point in your studies you have probably encountered notions like functions and limits. The exciting prospect is that there is much, much more to mathematics than this. In the broadest sense, mathematics concerns the study of patterns, connections, and logical structures. No matter the particular area of mathematics, the common goal is not only to understand the objects under study, but to relate them and form connections to other phenomena (mathematical and otherwise).

There are roughly two main branches of mathematics: pure and applied. This is not to say that each field of mathematics is sequestered to only one branch. Real analysis, for example, is a classically pure (“theoretical”) subject, yet it shows up in numerous applied areas of mathematics. Likewise, there are purely theoretical components to the major applied fields, such as statistics and differential equations. In the grand scheme of things, any differences in the pure and applied areas are unimportant: it is the content of the mathematics that matters.

Pure mathematics is more self-contained and formal. This is the kind of mathematics one studies “for its own sake.” Examples of pure areas are abstract algebra, discrete mathematics, and topology. The primary goal of a pure mathematician is to create new mathematics by proving theorems. A pure mathematician observes some common characteristics in the objects under study and formalizes the behavior by stating a conjecture. Then, by using a combination of prior knowledge and new insights, he or she attempts to prove the conjecture. If successful, the conjecture is elevated to the status of a theorem, and this is now added to the knowledge base of all of mathematics.

An applied mathematician does much of the same, but with a different objective and point of view. The subjects studied in the applied branch are inspired by other disciplines, such as biology, physics, finance, and computer science, just to name a few. An applied mathematician takes a real-world problem and attempts to model it with a suitable type of mathematics. This mathematical model is then analyzed, and the new information generated by the model must then be interpreted back to the original real-world setting. Along the way there are many issues that must be thoroughly considered, such as the appropriateness of the model and the validity of the conclusions it infers. This process often generates new and interesting mathematics by itself.

As you can see, whether pure or applied in focus, a great deal of mathematics is about creating and linking new ideas. This is where most mathematicians see the beauty of the subject. The further you go in your studies, the more interesting things get, so we encourage you to become very active and enthusiastic in your mathematics education. It will pay off.

1.2 Careers

A well-versed training in mathematics provides one with the ability to work in a variety of fields. Employers know that anyone with the ability to understand advanced mathematics has the capability to learn quickly and attack problems creatively. These are useful qualities
no matter the area of employment.

A mathematics background opens the door to an almost limitless number of career options. Several of the more common career paths are outlined below. You must remember that earning a degree in mathematics in no way requires you to enter a mathematically-oriented occupation in the strictest sense. More importantly, it is the general analytic and technical skills that you acquire as a mathematics major that will be your strong suit. Two of the very best repositories on mathematics-centric careers are housed by the American Mathematical Society and the Mathematical Association of America, the two leading professional organizations for mathematicians. Their careers websites are located, respectively, at the following links:

- [www.ams.org/profession/career-info/career-index](http://www.ams.org/profession/career-info/career-index)
- [www.maa.org/careers/](http://www.maa.org/careers/)

To see what recent UMW mathematics graduates are doing, browse our alumni profiles at

- [cas.umw.edu/math/alumni/](http://cas.umw.edu/math/alumni/)

You should also be aware that no one enters the work force with every bit of knowledge needed for their particular position. This is true no matter the occupation. For this reason, the most important skill one can have is the ability to grasp new situations and adapt quickly. The ability to assess complex problems critically and assimilate fine details into “the big picture” is crucial. Also, good technical writing skills are valued highly. Employers know that the person with a strong mathematics training will possess these important qualities.

### 1.2.1 Mathematics Education

In much the same way as literacy, general mathematical ability has always been a necessary “survival skill.” As technology becomes a more prominent part of our lives, the need for every person to understand basic mathematics is even more essential. This corresponds to a high demand for qualified mathematics teachers at the middle and high school levels. Moreover some states, including Virginia, now have “mathematics specialists” in the elementary schools.

In general, teaching can be a very rewarding career. If you enjoy discussing and thinking about mathematics, this may be a viable option for you. Try your hand at tutoring first and see if you can relate complex mathematical ideas to your pupil. If you are interested in teaching mathematics at the secondary level or being an elementary school teacher, you should apply to the UMW Teaching Education Program, part of the College of Education. Successful completion of this program includes courses and fieldwork beyond the major requirements and leads to licensure to teach in the Virginia public school system. Dr. Marie Sheckels is the education program coordinator in the mathematics department. For more information on the application process, see your UMW academic catalog.

### 1.2.2 Mathematics Education: The Collegiate Level

Teaching mathematics at a college or university is quite different from other educational settings. Naturally, the subjects that are taught are at completely different levels. However,
there is much more to being a professor than teaching a few classes each term. In particular, going into this profession requires much more than an enjoyment of teaching, for this is merely one component of the job. To be a successful professor, one must have a deep love of mathematics and the drive to create new ideas and connections.

First and foremost, it should be emphasized that a doctoral degree is mandatory for entering this profession at the university level. This in and of itself requires dedication and much hard work, with at least five years of serious mathematics training after the bachelor’s degree. A doctoral degree program culminates with the writing of a dissertation of original mathematics in a very specific field. Along the way you will have some course requirements (likely two or three years) and a series of rigorous exams. Beyond “traditional” mathematics, people with expertise in mathematics education are in great demand now, so a doctorate in this field is a highly marketable degree.

In addition to their regular teaching duties, a college professor is usually expected to contribute continually to their own research field and to the wider mathematical and university communities. The exact level of expectation is a function of the particular school. It takes an incredible amount of work to make a university run smoothly, and the faculty are responsible for a great deal of it outside the classroom.

Our program provides students the background required for success in graduate study not just in mathematics, but in related fields as well. In recent years we have had mathematics alumni earn doctoral degrees in mathematics, physics, statistics, and economics. If you do not wish to earn a Ph.D. you may want to think about a master’s degree as a viable alternative. This usually takes at most two years of full-time study to complete, a significantly lighter time commitment compared to the doctoral degree. With a master’s degree you would be well-qualified to teach at a community or junior college.

1.2.3 Government and the Private Sector

Planning and decision-making are fundamental components of much of the work in government and industry. In much the same way that equations predict behavior in physics, mathematical models are used to gain information about real-world phenomena. These models can be quite complex, and the mathematics involved goes much further beyond mere equations. Government agencies, as well as companies in the private sector, need someone who can understand the mathematical components of their projects.

An applied mathematician must understand all aspects of the problem at hand, not just the mathematics. For this reason, this type of work appeals to people that prefer to work on projects that stretch across several disciplines. The goal is to use mathematics to reveal patterns and make informed decisions and predictions on future behavior. For example, statistics may be used to understand the demographics of a rapidly changing neighborhood. The predictive power of statistics may be used to make important decisions regarding infrastructure: where to build new schools, how to structure the tax rates, where new roads should be constructed, etc. However, someone must interpret the results and ensure the validity and the reliability of the mathematical methods involved. This is the role of the mathematics expert.

Mathematical models are used in nearly every part of the private sector. Financial firms may use differential equations to predict trends in the markets. With the increasing role
that technology plays, security is a major concern of both the government and private firms. Discrete mathematics plays a large part in data security and reliability: there is even an entire field of mathematics devoted to this, known as information theory.

Actuarial work is an incredibly lucrative field now. Actuaries are mathematicians who specialize in probability and financial mathematics. For example, actuaries working in the insurance industry estimate the likelihood and cost of events and hence set premium rates in order to maximize revenue. Actuaries must pass a series of examinations to achieve professional designation as an Associate or Fellow, and students can begin taking these exams before graduation. Information about the actuarial exams and courses conducted by the Casualty Actuary Society and the Society of Actuaries is available at the following websites:

- www.casact.org/admissions/syllabus/
- www.soa.org/education/exam-req/

Students interested in becoming an actuary should strongly consider completing our minor in Actuarial Science. A great starting point for finding out more about the actuarial field is the website Be An Actuary, located at

- www.beanactuary.org

Working in the private sector requires some specialized mathematical knowledge. Many of the problems in government and industry concern vast amounts of data. Correctly interpreting this data is crucial. Therefore, a working knowledge of statistics is essentially mandatory. Also, understanding how measurable quantities change over time is critical, so a solid background in differential equations is essential. In addition to these purely mathematical subjects, some computational fluency is necessary. Some experience with programming is very attractive to these employers. Related to this is the field of numerical analysis, the study of computational routines that attempt to approximate solutions to complex mathematical problems.

In short, the odds are high that you can find rewarding work in a field that interests you, while making use of your mathematics training. If you are interested in working in a particular field, take some courses in that area. For instance, if you wish to work in finance, take a few business or economics courses. Not many people are experts in mathematics, and with a solid background you can be a valuable asset wherever you are employed. We have a strong track record of placing our graduates in stable jobs in government and industry. For examples of what recent mathematics alumni are doing in the private sector, visit our profiles page at

- cas.umw.edu/math/alumni/

1.3 The Mathematics Department at UMW

The department is centrally located on the main floor of Trinkle Hall. This is where you will find a majority of faculty offices, with a few on the lower level. Trinkle Hall functions as the hub of most of the activity in the department. Nearly all of our classes are held in
Trinkle, as well as the talks and some social gatherings sponsored by the department. The
department’s student computing lab is also found in Trinkle, in room B9 on the lower level.

A primary role of the department faculty is to teach your courses. A diverse array of
courses is offered each semester, covering introductory calculus courses all the way up to the
400 level. We encourage our majors to let us know if there are particular courses they would
like to have offered (however, we must be informed early). A department member may also
work closely with a student as a sort of mathematical mentor. If you have more focused
interests, you should talk to a faculty member about a directed study or honors project.

Beyond teaching classes, the department strives to create opportunities for student in-
teraction, both with the faculty and amongst the students themselves. We routinely plan
spring picnics, holiday gatherings, and hold a “Pizza and Problems” session a few times each
semester. Also, there are colloquium talks scheduled throughout the year, given mostly by
guests of the department. Students are encouraged to attend, and we try to invite some
students to have dinner with each speaker. All of these activities give the majors a chance
to meet each other, as well as the faculty they may not know from the classroom. For more
on student activities and involvement, see Section 4.
2 The Mathematics Major at UMW

2.1 Advisors

Declaring your major in mathematics is done with a simple paper form. When you declare your major the department chair will assign you a major advisor. The primary role of the faculty advisor is to help with the scheduling of your classes. This is accomplished by a face-to-face meeting when registration time nears each semester. Not only is this guidance helpful in planning a single semester’s layout, but the advisor is there to make sure that students are making sufficient progress towards graduation. Graduation requirements fall into three broad categories: general education, major requirements, and electives. Understanding the nuances of these various requirements takes some care, and meeting them requires much planning. Therefore the assistance your advisor provides is invaluable in making the most of your time here at UMW. It should be emphasized that the advisor is there to guide, but it is ultimately the student’s responsibility to ensure that all graduation requirements are met.

More generally, your advisor is someone that you should always feel free to approach when you have questions or concerns. Whenever you have an interest in mathematics outside the classroom—such as honors projects, independent studies, or department activities—your advisor will be happy to chat with you about such opportunities and point you in the right direction. In addition, students should always feel free to approach the department chair with any concerns related to their academic course of study.

2.2 Major Requirements

Other than prerequisite restrictions, there are no set-in-stone rules regarding the order in which the more advanced mathematics courses must be taken. There is, however, a standard track of courses leading to these upper-level classes.

Your highest priority is to complete the first two calculus courses as well as Math 201 in your first year at UMW. Many students earn credit for Math 121 through Advanced Placement, International Baccalaureate, or Cambridge Exams, and therefore start with Math 122, the second course in calculus. In this event, Calculus III (Math 223) should be taken in the first year. The basic rule of thumb is that you want to complete the four-course calculus sequence as soon as possible, and most students do so by the end of their sophomore year.

The major in mathematics requires a total of 36 credits, 15 of which must be from the following core classes:

- Math 223: Calculus III
- Math 224: Multivariable Calculus
- Math 300: Linear Algebra
- Math 431: Abstract Algebra I
- Math 471: Real Analysis I.
Linear Algebra is a prerequisite for most upper-level courses, so this course should be taken as early as possible; most students take it in their second year. Furthermore, Math 201 is a prerequisite for Math 300, which is why we suggest you complete it in the first year.

Three additional credits must be used to complete a 400-level course sequence. You may meet this requirement by taking one of Math 432 (Abstract Algebra II) or Math 472 (Real Analysis II). You may, of course, complete both sequences in algebra and analysis, applying the additional credits to the electives category in the major (described next).

In addition to the sequence requirement, there are 15 credits of upper-level electives required for the mathematics degree. Here you have quite a bit of freedom to choose the courses that interest you the most.¹ You may even count up to two independent studies (Math 491) toward the major in this category. These 15 credits of electives must satisfy the following rules:

- six credits must be at the 400 level, with at most three credits of directed study, and
- nine credits must be at the 300 or 400 level, with at most three credits of directed study.

So far we have 33 credits accounted for out of the 36 required. The remaining three credits may be earned from any of the following:

- any mathematics course numbered 207 or above,
- any computer science course numbered 220 or above,²
- any physics course numbered 105 or above,³ or
- Philosophy 306: Advanced Logic.

You have many options in completing the major electives. You should choose courses that best suit your current academic interests as well as your long-term career goals. Those students interested in working in the private sector are strongly encouraged to pursue courses in applied mathematics and statistics. This includes courses such as Differential Equations, Numerical Analysis, Probability and Statistical Inference, and Chaotic Dynamical Systems. Students interested in graduate work in mathematics should strongly consider taking Topology and Complex Variables. Finally, students in the UMW Teaching Education Program seeking certification to teach mathematics in grades six through 12 must also take Modern Geometry (Math 372). Otherwise, you should take the electives that most intrigue you—any exposure you can gain will benefit you in the long run.

We offer a wide range of courses with differing demands and audiences, and as a result not all courses run with the same frequency. You must keep this in mind as you are mapping out your path to graduation. Many of our courses are offered on a regular and predictable basis, and these are described in the table below. However, bear in mind that even this is not 100% certain and offerings may change depending on departmental resource and scheduling issues. When in doubt, contact the department chair.

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¹Internship credits in any discipline do not count toward the major.
²With the exception of CPSC 302.
³With the exception of PHYS 108.
## Course Offerings

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every semester</td>
<td>Math 201, 207, 223, 224, 300, 312, 431, 432, 471, 472</td>
</tr>
<tr>
<td>Once per year</td>
<td>Math 280, 321, 325, 330, 351, 352, 381, 382, 412, 441</td>
</tr>
<tr>
<td>Sporadically</td>
<td>Math 372, 411, 461, 481</td>
</tr>
</tbody>
</table>

On a final note, there is a limit on the number of credits in any one discipline you may count towards graduation. This cap of 60 credits should be kept in mind when planning for graduation. For example, there are 36 credits required for the mathematics degree. However, many students start their college mathematics courses with Math 121, Math 122, and Math 201. These courses are not required for the major, but act as prerequisites for the more advanced courses. Therefore, many students accumulate 45 credits in mathematics when working toward their degree. Moreover, many mathematics majors also take non-required courses such as Math 115 or Math 200. Adding in an independent study your senior year brings your total very close to the credit limit. In summary, think ahead, plan carefully, and talk to your advisor.

### 2.3 Graduating with Honors in Mathematics

The department offers mathematics majors the opportunity to graduate with honors in mathematics by completing an honors project. Of those students choosing to do so, most initiate the project during their senior year. An honors project entails an in-depth study of an area of mathematics not typically covered in the undergraduate curriculum. The choice of field is essentially left to the student and their honors advisor. The only stipulation is that the material must be of sufficient sophistication to warrant the honors designation, along with prior approval by the department faculty. Completion of an honors project requires both an expository report (the thesis) and an oral presentation to the department (the defense).

- **Requirements:**
  - Major in mathematics.
  - Minimum overall GPA of 3.0 and a minimum GPA of 3.5 in mathematics courses numbered 223 and above.
  - Completion of at least two 400-level mathematics courses (excluding 499) by the beginning of the approved honors activity.

- **To apply:**
  - Select a faculty member willing to supervise the honors project.
  - Write a project proposal with the aid of your honors supervisor, and present it to the department for approval.

After approval, the project will proceed under the guidance of the advisor. A suggested timeline for obtaining honors, from writing the proposal to graduation, can be found on the department’s website. Recent honors projects include:
• Michelle Craft, *Eigenvectors of Interpoint Distance Matrices* (2015), under Dr. Hydorn.


• Christopher Hunt, *Surface Theory, Minimal Surfaces, and Weierstrass Representations* (2014), under Dr. Chiang.

• Dane Lawhorne, *Analogies Between the Real and Digital Lines and Circles* (2014), under Dr. Helmstutler.


• Peter Slattery, *Analytical Approaches to the Wave Equation* (2013), under Dr. Lee.


• Marianne Dubinsky, *Probabilistic Solution to the Basel Problem* (2012), under Dr. Esunge.

• Alan Liddell, *Generalized Functions of Schwartz Type* (2012), under Dr. Collier.


• Kelly Scott, *Anti-blocking Sets* (2012), under Dr. Mellinger.

• Teresa Yao, *The Diffusion of a Chemical Pollutant Modelled by a Fourier Series* (2012), under Dr. Lee.

3 Courses in the Mathematics Department

3.1 Advanced Courses

The department has a rigorous program for students interested in studying serious mathematics. Our lower-level courses include foundational work in calculus (four semesters), as well as introductory courses in discrete mathematics and statistics. In addition to these courses aimed at serving the broader academic community, the department offers a wide variety of advanced courses at the 300 and 400 level. These courses are outlined here.

- **Math 300: Linear Algebra**
  Prerequisites: Math 122 and 201. Linear algebra is a complete study of linear equations, like $5x + 3y + z = 15$. Such equations naturally give rise to mathematical objects like matrices and vectors. The course includes topics such as matrix algebra, systems of equations, vector spaces, linear transformations, and eigenspaces. The techniques and methods of linear algebra are essential for advanced study in mathematics, and are useful in other fields, including chemistry, computer science (particularly computer graphics), and economics. Linear Algebra also serves as an introduction to theoretical mathematics, with the development of vector spaces from the axiomatic standpoint, and as such is an important bridge to 400-level courses in mathematics. Math 300 is required for the math major and is a prerequisite for Math 372, 431, 441, and 471. Math 300 can also be used as a prerequisite for Math 351.

- **Math 312: Differential Equations**
  Prerequisite: Math 122. This course is all about equations containing derivatives. For instance, given an equation of the form $y' + y = x^2y + 1$, can you determine the original function $y = f(x)$? A course in differential equations is a natural extension of the material in first and second semester calculus. Topics for this course may include linear and nonlinear differential equations, linear and nonlinear systems of equations, applications, approximations, and Laplace transforms. Differential equations describe how processes change and thus have wide applications in the real world. Many employers of mathematics majors expect job applicants to have experience working with data and real world problems as in Math 115, Math 200, Math 210, and Math 312. Math 312 is an elective in the mathematics and computer science majors, and it is a requirement for the physics major. Math 312 is encouraged for students studying physical chemistry as well. Ideally, this course would be taken soon after the completion of Math 122.

- **Math 321: Number Theory**
  Prerequisite: Math 201. Number theory is an elementary, theoretical study of properties of the integers. Topics usually include divisibility and prime factorization, linear and quadratic congruences, and congruences involving powers of integers. Number theory has a long and rich history, with some problems dating from the days of the Pythagoreans, but with many open questions inspiring ongoing research. A recent application of number theory is in the development of secure cryptography systems for networks of users, as in on-line financial transactions. Number Theory is an elective in
the math major, and is recommended for majors in preparation for the abstract algebra sequence. One section of Number Theory is typically offered each fall semester, and carries the Speaking Intensive designation.

- **Math 325: Discrete Mathematics**
  Prerequisite: Math 201. Discrete mathematics could be described as the mathematics behind computer science. In some sense, it is the opposite of calculus. In discrete mathematics we study arrangements of separated, or *discrete*, objects. The course may include topics such as discrete probability, graph theory, recurrence relations, topics from number theory, semigroups, and coding theory. Discrete mathematics is used extensively in modern computer science and has many applications as well as a rich theory. Many government agencies, like the National Security Agency, employ experts in discrete mathematics. This course is an elective in the major and serves as a good bridge course between the more computational courses like Calculus and the more abstract 400-level courses.

- **Math 330: Foundations of Advanced Mathematics**
  Prerequisite: Any mathematics course numbered 223 or higher. The goal of this course is to prepare you to study advanced mathematics courses, such as Abstract Algebra, Real Analysis, and Topology. You will learn how to analyze, construct, and write proofs, how to read mathematical text, understand definitions and theorems, and handle abstract concepts. The course includes topics such as basic logic, set theory, mathematical induction, relations, functions, sequences, cardinality, elementary number theory, and the axiomatic construction of the real numbers.

  Many students who did well in calculus courses experience difficulties in subsequent, more abstract, mathematics courses. This course, which students refer as the “proof” course, is designed to make this transition easier. You should take it as soon as you have the prerequisite.

- **Math 351–352: Numerical Analysis**
  Prerequisites: Math 223 and either Math 300 or Math 312. This elective course sequence covers the numerical methods that are applied to solutions of equations, interpolation, differentiation, integration, and solutions of differential equations and linear systems. These methods are appropriate for solving problems with no closed-form solution or where solutions are found iteratively. The topics covered in this course provide a good introduction to computational mathematics and its applications, including algorithms and programming. No previous experience with programming is required. Students who are planning on starting a career in applied mathematics upon graduation should consider taking this course. In the job market, having had some programming experience is helpful, particularly for jobs at Dahlgren where many of our graduates are employed.

- **Math 372: Modern Geometry**
  Prerequisite: Math 300. This course covers the axiomatic development of various geometries including modern Euclidean geometry, finite geometries, hyperbolic geometry,
and elliptic geometry. This amounts to examining what happens when we try to look at things like circles and triangles, but on a different surface. For instance, rather than looking at points and lines in a plane, we would discuss and analyze points and lines on a sphere. The course typically is taught using the dynamic software The Geometer’s Sketchpad which is used both to explore and discover theorems in Euclidean geometry. Applications of modern non-Euclidean geometries are also discussed. For example, finite geometry can be used to study things like statistical designs and cryptography. This is an interesting course in which you can prove results that contradict your intuitive notions of geometry. This course is not required for the major, but can be taken to fulfill one of the elective course requirements. Math 372 is required for those students certifying to teach secondary mathematics.

• Math 381–382: Probability and Statistical Inference
Prerequisite: Math 223. This elective course sequence provides an introduction to probability theory and calculus-based statistics, including probability distributions of discrete and continuous random variables, functions of random variables, and methods of estimation and statistical inference. The topics covered are those that form the mathematical foundation of statistical methods. No previous experience with statistics is required. With a total of nine credits in statistics, mathematics majors would be eligible for entry-level mathematics and statistics positions with government agencies, for example. Many graduate programs also require at least one course in calculus-based statistics.

• Math 411: Chaotic Dynamical Systems
Prerequisite: Math 223. This course covers function iteration, graphical analysis, periodic points, bifurcations, the transition to chaos, fractals, Julia sets and the Mandelbrot set. Dynamical systems has been one of the fastest growing fields of mathematics over the past several decades, as modern computer technology has made possible spectacular visualizations of the underlying theory. Applications of dynamical systems can be found in a wide variety of disciplines such as physics, biology, and economics. Math 411 is recommended for any mathematics major planning to pursue a career in government or industry.

• Math 412: Complex Variables
Corequisite: Math 471. The study of complex variables concerns the properties of the algebraic system that arises by adding the special number $i = \sqrt{-1}$. One can then discuss derivatives and integrals of functions of a complex variable. The course may include topics such as analytic functions, Cauchy-Riemann equations, contour integrals, the Cauchy integral theorem, and residues. Complex analysis is useful for its applications to physics, as well as forming the basis for some fractal generation, e.g. the Mandlebrot set. It also provides elegant and simple proofs of many formulas in trigonometry and calculus. The course is a popular mathematics elective and should ideally be taken after at least one semester of Real Analysis.
• **Math 431–432: Abstract Algebra**

Prerequisites: Math 300 and at least one other 300- or 400-level mathematics course. Abstract algebra is the study of structures such as groups, rings, and fields. These concepts generalize what you learned in high school algebra. You will still “add” and “multiply” but the objects will not necessarily be numbers and the operations will be defined in an abstract way in terms of their properties, such as associativity or commutativity.

Abstract algebra is a very rich subject, so one can study it for its own sake. However, its importance also lies in the fact that it provides powerful tools that are used in other areas of mathematics, for example in *algebraic* number theory and *algebraic* topology, as well as beyond mathematics, for example in physics, chemistry, and computer science.

You cannot study abstract algebra without writing proofs. Therefore, you should take some 300-level courses to prepare for this class. In addition to Math 300, which is a prerequisite, good courses to take are Math 321, Math 325, and Math 330. Math 431 is required for the major, while Math 432 is an elective which would satisfy the sequence requirement.

• **Math 441: Topology**

Prerequisites: Math 300 and at least one other 300- or 400-level mathematics course. Roughly speaking, topology is an abstract study of geometry. Essentially, one studies the general configuration of geometric objects without regard to any rigid structure or measure. The topics in Math 441 include the fundamental concepts of point-set topology: continuity, connectedness, compactness, products and quotients, and the separation axioms.

Topology is a relatively new field of mathematics, and topological methods arise in several major fields of mathematics including algebra, analysis, and differential equations. More recently, topology has been used by physicists to describe the shape of space and by chemists to study the structure of DNA and proteins. As the field of topology generalizes portions of real analysis, it may be a good idea to take Math 471 before Math 441.

• **Math 461: Topics in Mathematics**

Prerequisite: Course dependent. The content and scope of this course depends on the instructor and the topic. Recent topics have included financial mathematics, partial differential equations, and coding and cryptography. May be taken up to three times for credit.

• **Math 471–472: Real Analysis**

Prerequisites: Math 223, Math 300, and at least one other 300- or 400-level mathematics course. Real Analysis is a rigorous development of the methods of calculus. This course may include topics such as sequences and series of real numbers, metric spaces, continuity of functions, connectedness, completeness, compactness, measure zero, Riemann integration, and sequences of functions. Real analysis serves as a firm basis
for numerical analysis, complex analysis, and functional analysis with applications to quantum physics. Ideally, this course sequence should be started after a semester of Abstract Algebra, but before Topology. Math 471 is required for the major, while Math 472 is an elective which would satisfy the sequence requirement.

- **Math 481: Theory of Interest**
  Prerequisite: Math 223. This course introduces the mathematical concepts underlying the theory of interest. Topics include measurement of interest (including accumulated and present value factors), annuities, yield rates, amortization schedules and sinking funds, bonds and related securities, derivative instruments, and hedging and investment strategies.

- **Math 491–492: Directed Study**
  Prerequisite: Departmental permission. Math 491 is an individual study under the direction of a faculty member, beyond the scope of our normal course offerings. Independent studies may lead to graduation with honors in mathematics. See Section 3.2 for more information.

- **Math 499: Internship**
  Supervised off-campus experience, developed in consultation with the department. Internship credits do not count toward the major.

### 3.2 Independent Study Opportunities – Math 491

The department offers students the opportunity to explore more advanced topics through an independent study, which could have the added benefit of leading to an honors project. Students interested in completing an independent study should consult with a faculty member who is interested in directing a student in the desired field. Here is a list of faculty and their independent study interests.

- **Yuan-Jen Chiang** has worked with students on the following topics: topics in analysis, differential geometry, tensors and relativity, partial differential equations, introduction to harmonic maps, applications of mathematics in electrical engineering, fractional calculus, topics in Euclidean and non-Euclidean geometries, tensor analysis in physics and engineering, Fourier analysis, and minimal surfaces.

- **Gary Collier** has led directed studies in generalized function theory.

- **Melody Denhere** is interested in working with students on problems in probability and statistics. Future projects may include simulation studies, statistical modelling and analysis of real world problems in preparation for careers in mathematics and statistics. Melody is also keen to direct studies in big data analytics, biostatistics, regression analysis, multivariate data analysis, robust statistical methods, and outlier analysis.
• Julius Esunge welcomes the opportunity to work with students on problems in probability, statistics and financial mathematics. Examples of possible topics include the following: simulation of random variables and probability distributions; properties and applications of Brownian motion, Brownian bridge and fractional Brownian motion; applications of limit theorems in probability; applications of stable distributions for heavy tailed data; statistical analysis and properties of actuarial models. Julius also enjoys directing reading courses/seminars towards actuarial examination preparation.

• Randall Helmstutler has directed students in digital topology, homotopy and Lie theory, category theory, advanced group theory and linear algebra, and cryptology. His general interest is in working with students wishing to delve into advanced topics in abstract algebra and topology. In the field of algebra this includes group theory, ring theory, and applications of group theory to cryptographic protocols. Topologically, he would enjoy directing studies in homotopy and covering spaces, differential topology, and topological group theory. To see what students have done under Dr. H., visit his undergraduate research page at

   doctorh.umwblogs.org/student-research/

• Debra Hydorn is interested in working with students on research projects in probability and statistics, including probability simulations and statistical modelling. She is particularly interested in GIS, biostatistics and environmental applications. She has led directed studies on regression analysis, linear models, statistical computing, education methods for statistics, preparation for actuarial exams, and for Math 351–352 and Math 381–382 during semesters in which these courses have not been offered.

• Janusz Konieczny has directed students in the study of field theory, topics in geometry, applications of linear algebra, and semigroup theory.

• Jangwoon (Leo) Lee is interested in working with students in various areas of applied mathematics including partial differential equations, scientific computations, optimal control problems, and numerical methods for mathematical model equations such as stochastic/partial differential equations.

• J. Larry Lehman has led directed studies in Galois theory, algebraic number theory, and elliptic curves. He would also be interested in working with students in analytic number theory and other advanced topics in number theory.

• Keith Mellinger has directed students in various areas of discrete mathematics including coding theory, design theory, finite geometry, combinatorics, graph theory, and cryptography. He also directed a joint project with a student and a Navy scientist as part of the Dahlgren Research Collaboration. Dr. Mellinger is interested in working with students in any of these areas and more information about his projects with students can be found on his Student Research webpage, located at

   www.keithmellinger.com/student-research/
• Suzanne Sumner has worked with students on mathematical modelling, fractal geometry, dynamical systems, applications in finite mathematics, operations research, epidemiological models, and differential equations.
4 Opportunities for Our Students

4.1 Undergraduate Research

The department actively supports those students interested in doing original research. If you are interested in summer undergraduate research programs (REUs) in mathematics sponsored by the National Science Foundation or other organizations, the following websites are valuable resources:

- www.nsf.gov/crssprgm/reu/
- www.maa.org/programs/students/undergraduate-research

4.1.1 The UMW Summer Science Institute

Every summer, the University of Mary Washington conducts a research program for its undergraduates. This 10-week program has faculty-selected students working together on research projects chosen by the participating faculty member. The Summer Science Institute concludes with a symposium where students give presentations and display posters of their work. The program includes a stipend and free housing and board for the students. If you are interested in the SSI program you should contact the faculty member with whom you are interested in working. Past projects in our department include:

- Rebecca Revercomb, *Yield Curves and Interest Rate Modeling* (2015), under Dr. Esunge.
- Dane Lawhorne, *Analogies Between the Real and Digital Lines and Circles* (2013), under Dr. Helmstutler.
• Kwadwo Brobbey and Benjamin Tuxbury, *Stochastic Programming and Optimization* (2012), under Dr. Esunge.

• Morgan Brown, *How to Win Every Time* (2012), under Dr. Lee.

• Peter Slattery, *Brave the Wave* (2012), under Dr. Lee.


• Kathryn Dillinger, *Profiting with Options Using the Black-Scholes Equation* (2011), under Dr. Lee.


• Teresa Yao, *Concentration of a Chemical Pollutant Modelled by a Fourier Series* (2010), under Dr. Lee.

• Kevin Groat and Andrew Snyder-Beattie, *Geometric Brownian Motion, a Safe Assumption?* (2010), under Dr. Esunge.

• Sarah Ball, *A Comparison of Remedial Measures for Multicollinearity in Multiple Regression Analysis* (2009), under Dr. Hydorn.


• Kathryn Christian, *An Exact Solution to Laplace’s Equation Inside a Rectangle* (2009), under Dr. Lee.

• Barbara Brown, *Generalized Dihedral Groups and Geometry* (2009), under Dr. Helmstutler.

• Jonathan Stallings, *Improved Confidence Ellipses in GIS* (2008), under Dr. Hydorn.


• Catherine Castleberry and Katie Hunsberger, *Coding and Cryptography with Hyperovals of PG(2,2^s)* (2008), under Dr. Mellinger.
4.1.2 Research During the Academic Year

Of course, students are also encouraged to approach professors about the possibility of research projects during the regular semesters. This has the added benefit of potentially leading to an honors project. The university supports such endeavors through undergraduate research grants, which may be used to purchase supplies or to fund travel to professional conferences. The mathematics faculty have a wide array of interests and can direct projects in many areas: see Section 3.2 above for a summary of faculty interests.

Students interested in careers in the private sector are strongly encouraged to consider the department’s Dahlgren Research Collaboration program, where teams of students are matched to working Navy scientists on real problems in mathematics, computing, and data analysis. Interested students should seek out Dr. Debra Hydorn, the primary point of contact for the program, for additional information.

4.2 Professional Events

The department hosts a fall speaker series each year, where we invite mathematicians from other schools and businesses to speak about their involvement in the mathematical community. In the spring we sponsor guest lecturers in conjunction with the department’s Mathematics Awareness Month activities. Recent speakers have come from government industries such as the National Geospatial-Intelligence Agency, the Naval Surface Warfare Center at Dahlgren, and the National Security Agency. We also invite mathematicians from other universities, including Virginia Tech, James Madison University, and the University of Richmond. We usually treat each guest speaker to dinner, and we like to include some students in each outing.

4.3 Social and Scholastic Opportunities

The mathematics majors at UMW formed their own student section of the Mathematical Association of America. They hold regular meetings where they discuss semester activities and plan events. Every year, the students design a t-shirt that features a witty mathematical expression, and these are made available for purchase. The MAA student group has played a very active role in the annual UMW Calculus Tournament for area high school students and is always looking for new members to become involved in all their activities.

The department offers regular social activities for the mathematics majors. Each March we celebrate “Pi Day” (can you guess the date?) with an abundance of pie, and our MAA student group sponsors our annual holiday party and the spring picnic. In conjunction with Mathematics Awareness Month, the department offers a career night in the spring to help students start thinking about their career options. In the fall of 2005 the department started a “Pizza and Problems” session that runs several times each semester, where students enjoy a slice of pizza while socializing and discussing some interesting mathematical puzzle. Our majors are typically very active and work hard to offer a variety of social activities for their peers and the faculty.

The department also sponsors a local chapter of Pi Mu Epsilon (ΠME), which is a national honor society in mathematics. Student members receive a subscription to the ΠME journal.
upon induction. Furthermore, the department sponsors the Oscar T. Schultz Award, which is a cash prize given to an outstanding junior or senior mathematics major.

With the Chair’s List, the department also honors students on a semester basis for their academic achievements. Students who maintain a minimum GPA of 3.5 for their mathematics courses are named to the Chair’s List. Those earning this honor are recognized by the department chair through an announcement to the department faculty and in the department’s display case in the hallway by Trinkle 140. Inclusion into the Chair’s List is open only to declared mathematics majors who have completed at least four courses required for the major.

Together with donors, the department has established four scholarships:

- The Meredith C. Loughran ’94 Scholarship (open to all students, with preference given to mathematics majors)
- The Louise W. Robertson, M.D. ’56 Scholarship (open to students majoring in mathematics or a health-related field)
- The Mary Farley Talley ’66 Scholarship (open to mathematics majors)
- The Merrilyn Sawyer Dodson ’68 scholarship (open to rising junior or senior mathematics majors).

Selection of recipients shall be made by the Office of Financial Aid under the recommendation by a committee of faculty within the department. Students interested in applying for the Loughran Scholarship should contact Dr. Suzanne Sumner in the mathematics department.

4.4 Mathematics Department Aides

The department employs up to five students per semester to act as aides for the faculty. The aides assist faculty in photocopying, grading for lower-level courses, and basic office needs. Occasionally, students interested in teaching are assigned teaching apprentice duties by faculty members. Interested students should contact Joyce Durham, our office manager.
5 The Faculty and Their Interests

The faculty in the Department of Mathematics at UMW come from a wide array of backgrounds in various fields of expertise. This blend of abilities adds to the strength of the department. Some background information on the mathematics faculty follows.

- **Yuan-Jen Chiang**, Professor
  - Trinkle 124, 654-1326, ychiang@umw.edu

  Dr. Yuan-Jen Chiang earned a Ph.D. from Johns Hopkins University. She has done research and published articles in a variety of areas, including recently seeing the publication of her book *Developments of Harmonic Maps, Wave Maps and Yang-Mills Fields into Biharmonic Maps, Biwave Maps and Bi-Yang-Mills Fields*. Dr. Chiang has been a reviewer for Mathematical Reviews, published by the American Mathematical Society, and has written commentaries for approximately 100 scholarly papers on differential geometry and global analysis. She regularly teaches our courses on Real Analysis (Math 471–472), Topology (Math 441), Complex Variables (Math 412), Differential Equations (Math 312) and Calculus.

- **Manning G. Collier**, Associate Professor
  - Trinkle 122, 654-1327, gcollier@umw.edu

  Dr. Manning G. Collier earned the Ph.D. in mathematics from Vanderbilt University. Dr. Collier is an expert in general mathematics, real and complex analysis, and functional analysis. He regularly teaches the Real Analysis sequence (Math 471–472), Topology (Math 441), and the Complex Variables elective (Math 412), as well as Calculus.

- **Melody Denhere**, Assistant Professor
  - Trinkle B45, 654-2162, mdenhere@umw.edu

  Dr. Melody Denhere earned a Ph.D. in mathematics and an M.S. in applied mathematics from Auburn University in Alabama. Her area of specialty is functional data analysis and robust statistical methods. Dr. Denhere regularly teaches our probability and statistics classes (Math 200, Math 280, and Math 381–382), Numerical Analysis (Math 351–352), and Calculus.

- **Julius Esunge**, Associate Professor
  - Trinkle B49, 654-2028, jesunge@umw.edu

  Dr. Julius Esunge holds a Ph.D in mathematics from Louisiana State University in Baton Rouge. His expertise is in probability, statistics and stochastic analysis, where he specializes in stochastic differential equations. His teaching interests include Actuarial and Financial Mathematics, Real and Complex Analysis, Probability, Statistics and Calculus.
• **Randall D. Helmstutler**, Chair and Associate Professor

  – Trinkle 128, 654-1329, rhelmstu@umw.edu

Dr. Randall Helmstutler (a.k.a. “Dr. H”) earned the Ph.D. in mathematics from the University of Virginia. He is a trained topologist whose interests lie primarily in algebraic topology and category theory, and he has directed numerous undergraduate research projects in these fields. He regularly teaches Topology (Math 441), the Abstract Algebra sequence (Math 431–432), Multivariable Calculus (Math 224), and the first-year seminar in Cryptology.

• **Debra L. Hydorn**, Professor

  – Trinkle 132, 654-1330, dhydorn@umw.edu

Dr. Debra L. Hydorn earned a Ph.D. in statistics from the University of Michigan. Her areas of interest include methods in multivariate statistics, statistics and mathematics education, computational statistics, service-learning, statistical consulting, and interdisciplinary teaching. Dr. Hydorn regularly teaches courses in Probability and Statistical Inference (Math 200, Math 280, Math 381, and Biol 260), Numerical Analysis (Math 351), Mathematical Modeling (Math 115), Introduction to Discrete Mathematics (Math 201) and the first-year seminars EscherMath and Infographics.

• **Janusz Konieczny**, Professor

  – Trinkle 125, 654-1331, jkoniecz@umw.edu

Dr. Janusz Konieczny (a.k.a. “Dr. K”) earned a Ph.D. in mathematics from Penn State University. Among his honors are the university’s Outstanding Young Faculty Member Award and Waple Professorship. Dr. Konieczny is an expert in the field of algebraic semigroups, where he has published many research articles in refereed journals. He is also a regular referee and reviewer for research journals. Dr. K regularly teaches our Abstract Algebra courses (Math 431–432) as well as Topology (Math 441), Linear Algebra (Math 300) and Calculus.

• **Jangwoon “Leo” Lee**, Associate Professor

  – Trinkle 126, 654-2026, llee3@umw.edu

Jangwoon “Leo” Lee earned a Ph.D. in applied mathematics from Iowa State University. He is an expert in stochastic partial differential equations and numerical analysis, where he has published research articles in refereed journals. He has served as a referee for several research journals, as a judge for the Mathematical Association of America undergraduate poster session, and as an exam reviewer for the National Math and Science Competition for students from 4th to 11th grade in schools in the US and Canada. He regularly teaches Real Analysis (Math 471–472), Numerical Analysis (Math 351), Differential Equations (Math 312), and Calculus.
• J. Larry Lehman, Professor

  – Trinkle 130, 654-1332, llehman@umw.edu

Dr. Larry Lehman earned a Ph.D. from the University of Virginia. Dr. Lehman is an expert on algebra and number theory where he has published several articles. He regularly teaches our Abstract Algebra sequence (Math 431–432) and Number Theory (Math 321), as well as Linear Algebra (Math 300) and Calculus.

• Jennifer Magee, Lecturer

  – Trinkle B50, 654-1431, jmagee@umw.edu

Jennifer Magee holds an M.A. in mathematics from Bowling Green State University. Her mathematical interests lie in algebra and pedagogy. Ms. Magee primarily teaches 100-level course offerings for the department.

• Keith E. Mellinger, Professor

  – Trinkle 129 and ITCC 131, 654-1333, kmelling@umw.edu

Dr. Keith E. Mellinger earned a Ph.D. in mathematics from the University of Delaware and was a post-doctoral fellow at the University of Illinois at Chicago before coming to UMW in 2003. Dr. Mellinger’s research articles have been published in several journals. He is an expert in discrete mathematics, error-correcting codes, and finite geometry, and he received a research grant in 2005 from the National Security Agency to continue his work in these fields. One of his articles in Mathematics Magazine was recognized in 2010 with one of the national writing awards from the Mathematical Association of America. In addition to his teaching and professional work, Dr. Mellinger currently serves as the Director of the First-Year Seminar Program. He regularly teaches the advanced Discrete Mathematics course (Math 325), Modern Geometry (Math 372), Calculus and first-year seminars.

• Kelly Perkins, Lecturer

  – Trinkle B41, 654-1328, kperkins@umw.edu

Kelly Perkins earned his Bachelor’s degree in Mathematical Sciences and Economics at Rice University in Houston. He also holds a J.D. from the University of Houston and an M.S. in Mathematical Sciences with a concentration in statistics from Virginia Commonwealth University. He has worked in the actuarial profession and as an attorney. He earned a Virginia teaching license, taught algebra and statistics, and is a member of the National Council of Teachers of Mathematics. Mr. Perkins primarily teaches general education courses in mathematics and his interests include teaching an appreciation and understanding of math concepts to non-math majors.
• **Marie P. Sheckels**, Professor
  
  – Trinkle 123, 654-1334, msheckel@umw.edu

Dr. Marie Sheckels earned a Ph.D. in mathematics education from the University of Maryland. A member of the National Council of Teachers of Mathematics, Dr. Sheckels is an expert in mathematics education. She and her colleagues have received grants from the National Science Foundation, the State Council of Higher Education in Virginia and the Fund for the Improvement of Postsecondary Education. She has recently been involved in investigating the roles and effectiveness of mathematics specialists in elementary schools, and she is currently the director for the M.Ed. in Secondary Mathematics program for in-service teachers.

• **Suzanne Sumner**, Professor
  
  – Trinkle 131, 654-1335, ssumner@umw.edu

Dr. Suzanne Sumner earned a Ph.D. and M.S. in applied mathematics from North Carolina State University, after receiving a B.S. in mathematics from the University of North Carolina at Chapel Hill. A member of Phi Beta Kappa, Dr. Sumner is an expert on dynamical systems. The recipient of teaching awards at North Carolina State University and Mary Washington, she is active in efforts to improve teaching innovation and to promote interdisciplinary connections. She teaches our History of Mathematics course (Math 207) and Chaotic Dynamical Systems (Math 411) as well as Differential Equations (Math 312) and Mathematical Modelling (Math 115). Dr. Sumner also teaches the first-year seminars The Mathematics of Chaos (FSem 100D) and Race & Revolution: James Farmer & the Struggle for Civil Rights (FSem 100G4).