

Final Report for 2009-2010 Faculty Research Grant

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I received a 2009 - 2010 Faculty Research Grant of \$3,500 as a summer stipend for the project “*Finite Element Approximations of Stochastic Neumann Boundary Value Problems*”. In this project, my main goal was to analyze a stochastic mathematical model equation for transport of pollutants in groundwater under the Neumann boundary condition (for my model, see Section 1 of my proposal or Section 1 of the attached paper).

As I mentioned in Section 2 of the proposal, I showed the existence and uniqueness of the solution to the stochastic model (see Section 3.3 of the paper); I utilized the analytical tool called the Karhunen-Loeve expansion to express stochastic input data in terms of eigenfunctions (Section 2 of the paper); I transformed my stochastic model into a deterministic model with finite dimensional information (Section 4.1 of the paper); I used the finite element method to estimate errors of the solution to the high-dimensional problem (Section 4.4 of the paper).

Fortunately, I was able to get more results from “*Optimization of the Model*” that is an application of the stochastic problem and can be considered as another high-level project in my area. Throughout this additional work, I proved the existence of an optimal solution to the optimization problem (Section 5.2 of the paper); I used the method of Lagrange multipliers to derive the optimality system of equations (Section 6.4 of the paper); I approximated an optimal solution and estimated errors between the exact and computational optimal solutions using the finite element method (Section 7.3).

As I anticipated, I proved new theorems about my stochastic model and developed a discrete solver for the model. I plan to submit the paper to a referred applied mathematics research journal soon. I believe my result on this project will add a new class of methods to the literature for solving stochastic mathematical model equations using the transformation from stochastic problems into deterministic problems. I gave two presentations about my project in international-level conferences in Korea and USA. I used my results as examples for my numerical analysis course and PDE course. Also I plan to use a simple and similar version of this project by dropping random terms for undergraduate students in SSI 2010.