

RANDALL HELMSTUTLER
DEPARTMENT OF MATHEMATICS
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This project concerned an effort to unify several disparate techniques in algebra and topology, all falling under the umbrella term *idempotent splitting*. Methods of idempotent splitting are standard in the various branches of algebra (and have been for probably a century now), but similar techniques have only recently entered topology, most prominently in *stable homotopy theory*. The goal of the project was to use ideas of *model category theory* to better understand idempotent splittings in stable homotopy and to give a general framework in which the algebraic and topological methods could be viewed merely as different instances of the same technique.

The first part of my work dealt with generalizing the Hovey-Palmieri-Strickland Theorem, probably the most general topological result on idempotents having far-reaching consequences. The HPS result has the advantage of already being phrased in the language of category theory. However, in its original published form, the statement of HPS does not lend itself to any sort of comparison with algebraic methods. As part of this project, I was able to show that the notion of a *semi-stable model category* was sufficient to capture the HPS result, the advantage being that such phrasing is also adaptable to the various fields of algebra.

The main difficulty of obtaining this result is that HPS is a theorem on *homotopy categories*, which have a highly non-trivial relation to model categories. Thus a bulk of my work was to understand how to translate the homotopy-theoretic arguments of HPS to the level of model categories. I was able to show that the structure of semi-stable model categories allows one to prove the HPS result using only established techniques of model categories. This result was previously unknown.

In this way, we have that the HPS result on idempotent splitting is really a statement about a special kind of model category. The upshot is that the same type of category occurs in algebra. In fact, I was able to prove that the various types of idempotent decomposition in algebra all occur in the same way. Hence, the methods of idempotent splitting in semi-stable model categories capture and unify such techniques in both algebra and topology simultaneously.

While successful in this regard, these results are not publishable without new examples (as already pointed out in a referee report for another article of mine on semi-stable model categories). The structure of model categories is tight enough to cause them to be rare, and imposing extra conditions like semi-stability only exacerbates the problem. While I have given a unification of these results on idempotents, I have not been able to find any new examples or results that are not currently in the literature. In other words, my proofs are unique but my results are not (so far). I am currently working on fixing this, as finding more examples of such categories would remedy problems with two of my current projects at once.